

# The Entropic Dynamics of Relativistic Quantum Fields\*

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## Abstract

The formulation of quantum mechanics within the framework of entropic dynamics is extended to the domain of relativistic quantum fields. The result is a non-dissipative relativistic diffusion in the  $\infty$ -dimensional space of field configurations. On extending the notion of entropic time to the relativistic regime we find that the field fluctuations provide the clock that sets the scale of duration. We also find that the usual divergences that affect all quantum field theories do not refer to the real values of physical quantities but rather to epistemic quantities invariably associated to unphysical probability distributions such as variances and other measures of uncertainty.

## 1 Introduction

The goal of the Entropic Dynamics (ED) framework is to seek for quantum mechanics a level of understanding comparable to that attained by Jaynes in statistical mechanics and thermodynamics. The challenge is graphically described by Jaynes' omelette metaphor: "Our present QM formalism is a peculiar mixture describing in part realities in Nature, in part incomplete human information about Nature — all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble." [1]

In previous work the Schrödinger equation for the non-relativistic quantum dynamics of  $N$  particles was derived as an example of entropic inference.[2] Within the ED framework the ontic and the epistemic elements of the model are neatly unscrambled.<sup>1</sup> The non-relativistic model proposed in [2] has a clear ontology of particles with real and definite, albeit uncertain, positions. This is

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<sup>1</sup>The distinction ontic/epistemic is not to be confused with the distinction objective/subjective: a probability is a purely epistemic notion that incorporates both subjective and objective elements. Indeed, on one hand, the assignment of priors and likelihoods involves judgments that are inevitably subjective and, on the other hand, the very reason why

in stark contrast with the standard interpretation of quantum mechanics which refrains from attributing definite values to any observable prior to an actual measurement. The wave function and other “observables” such as momentum and energy do not represent anything physically real. They reflect epistemic rather than ontic aspects of the model. They are properties associated to the probability distributions and not to the particles themselves.[4][5]

But for such an epistemic view of quantum states to be satisfactory it is not sufficient to accept that  $|\psi|^2$  represents a state of knowledge; all other features of the wave function must be epistemic too. One must provide an epistemic interpretation for the phase of the wave function and, furthermore, one must show that changes or updates of the epistemic  $\psi$  — including both unitary time evolution according to the Schrödinger equation and the projection postulate during measurement — occur precisely according to the stipulations of entropic and Bayesian inference.[6] The ED framework has led to several new insights including the entropic nature of time [3]; an entropic interpretation of the phase of the wave function and of gauge transformations; the role of Hilbert spaces and complex numbers; the uncertainty relations [4]; and the quantum measurement problem and the interpretation of “observables” other than position [5].

In this paper we take the first step towards extending entropic dynamics to the domain of relativistic scalar quantum fields. The basic dynamical quantity, the probability of a small change, is found maximizing the appropriate entropy subject to suitable constraints. Then the notion of time is introduced to keep track of the accumulation of small changes. The resulting evolution is described by a functional Fokker-Planck equation and a (quantum and relativistic) functional Hamilton-Jacobi equation. These equations are then combined into a functional Schrödinger equation which is formally equivalent to the standard quantum theory of scalar fields (see *e.g.*, [7]).

Many of the standard predictions follow immediately. For example, the excited states of the field can be interpreted as identical particles that obey Bose-Einstein statistics. We also find the usual infinities that plague all quantum field theories but with one major difference: the divergences do not refer to the real values of physical quantities but rather to unphysical epistemic quantities such as variances and other measures of uncertainty. Thus, in entropic dynamics the infinities of quantum field theory are not physical effects; they are indications that the information that has been included in the analysis is insufficient to answer certain questions. This result supports Jaynes’ intuition foreseeing “...the possibility of a future quantum theory in which the role of incomplete information is recognized ... when we free ourselves from the delusion that probabilities are physically real things, then when [a dispersion]  $\Delta F$  is infinite, that does not mean that any physical quantity is infinite. It means only that the theory is completely unable to predict  $F$ . The only thing that is infinite is the uncertainty of the prediction.”[1]

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we collect data and other information is precisely in order to update probabilities and thereby enhance their objectivity.

## 2 Entropic dynamics

We are concerned with the dynamics of a single scalar field. The generalization to other Boson fields is immediate. A particular field configuration  $\phi(x)$  associates one degree of freedom to each spatial point  $x$  in three dimensional Euclidean space. Such a field is represented as a point  $\phi \in \mathcal{F}$  in the  $\infty$ -dimensional configuration space  $\mathcal{F}$ . Our first assumption is that the space  $\mathcal{F}$  is flat and its metric is a straightforward generalization of the metric  $\delta_{ij}$  of Euclidean space so that the distance  $\Delta\ell$  between two slightly different configurations  $\phi$  and  $\phi + \Delta\phi$  is written as

$$\Delta\ell^2 = \int d^3x [\Delta\phi(x)]^2. \quad (1)$$

The justification of this assumption and of several others that will follow is, in the end, purely pragmatic. An analogy can be drawn to other physical theories. For example, in Newtonian mechanics different physical situations are described by different forces; in an entropic framework different physical situations are described by different constraints. (We shall later see that  $\Delta\ell$  will enter as a constraint.) And just as Newtonian mechanics did not justify the  $1/r^2$  force law except through its empirical success, we will not, at this early point in the development of the ED of quantum fields, offer any deeper insight into the choice of distance except to note that it is empirically successful too.

The second assumption is that in addition to the field  $\phi$  the world contains other stuff described by variables  $y$  living in some space  $\mathcal{Y}$ .<sup>2</sup> The number and nature of the extra variables  $y \in \mathcal{Y}$  need not be specified; we only need to assume that their values are uncertain and that this uncertainty is described by some probability distribution  $p[y|\phi]$  that depends on the particular field configuration  $\phi$ .<sup>3</sup>

The  $\infty$ -dimensional manifold of distributions  $p[y|\phi]$  — for each field configuration  $\phi$  there is a corresponding  $p[y|\phi]$  — is a *statistical manifold*  $\mathcal{M}$  and each distribution  $p[y|\phi] \in \mathcal{M}$  can be conveniently labeled by its corresponding  $\phi$ . For future reference, the entropy  $S[\phi]$  of  $p[y|\phi]$  relative to an underlying measure  $q[y]$  of the space  $\mathcal{Y}$  is given by the functional integral

$$S[\phi] = - \int Dy p[y|\phi] \log \frac{p[y|\phi]}{q[y]}. \quad (2)$$

The dynamics follows from yet a third assumption, that large changes result from the accumulation of many small changes. Thus, the basic dynamical problem is to find the transition probability  $P[\phi'|\phi]$  of a small change from an initial  $\phi$  to a new  $\phi' = \phi + \Delta\phi$ . Since neither the new field  $\phi'$  nor the new  $y'$  are known the relevant space is  $\mathcal{F} \times \mathcal{Y}$  and we seek the joint distribution  $P[\phi', y'|\phi]$ .

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<sup>2</sup>In statistics such variables are called *nuisance* variables. Although we are not directly interested in them they affect the variables we do care about and must be included in the analysis.

<sup>3</sup>The notation we adopt is standard in theoretical physics: the  $x$ -dependence is denoted as a subscript,  $\phi(x) = \phi_x$ ; square brackets as in  $F[\phi]$  denote functionals; functional derivatives are written  $\delta/\delta\phi_x$ ; and functional integrals are written as  $\int D\phi F[\phi]$ .

This is found maximizing the (relative) entropy,

$$\mathcal{S}[P, Q] = - \int D\phi' Dy' P[\phi', y'|\phi] \log \frac{P[\phi', y'|\phi]}{Q[\phi', y'|\phi]}, \quad (3)$$

subject to constraints that codify the appropriate relevant information.

**The prior:** We assume a state of extreme ignorance represented by a product,

$$Q[\phi', y'|\phi] = q[\phi']q[y'], \quad (4)$$

where  $q[\phi]$  and  $q[y]$  are uniform distributions.

**The first constraint:** Use the product rule to write  $P[\phi', y'|\phi] = P[\phi'|\phi]P[y'|\phi', \phi]$ .

The first factor  $P[\phi'|\phi]$  is the transition probability we want to find. The second factor,  $P[y'|\phi', \phi]$ , is constrained to remain on the manifold  $\mathcal{M}$ , that is  $P[y'|\phi', \phi] = p[y'|\phi'] \in \mathcal{M}$ . Therefore,

$$P[\phi', y'|\phi] = P[\phi'|\phi]p[y'|\phi']. \quad (5)$$

This constraint is implemented by direct substitution into the entropy (3):

$$\mathcal{S}[P, Q] = - \int D\phi' P[\phi'|\phi] \log \frac{P[\phi'|\phi]}{q[\phi']} + \int D\phi' P[\phi'|\phi] S[\phi'] \quad (6)$$

where  $S[\phi']$  is given by eq.(2).

**The second constraint** represents the fact that physical changes are not discontinuous: the requirement that  $\phi'$  be an infinitesimally close to  $\phi$  is implemented by imposing that the expectation

$$\langle \Delta \ell^2 \rangle = \int D\phi' P[\phi'|\phi] \int d^3x (\Delta \phi_x)^2 = \Delta \lambda^2[\phi] \quad (7)$$

take some infinitesimal but for now unspecified value  $\Delta \lambda^2$  which could depend on  $\phi$ .

Maximizing  $\mathcal{S}$  subject to the constraints above plus normalization yields,

$$P[\phi'|\phi] = \frac{1}{\zeta} \exp \left( S[\phi'] - \frac{1}{2} \alpha[\phi] \int d^3x (\phi'_x - \phi_x)^2 \right), \quad (8)$$

where  $\zeta$  is a normalization constant and  $\alpha[\phi]$  is the Lagrange multiplier that implements the constraint (7). For small  $\Delta \phi$  which corresponds to large  $\alpha$ , we can expand

$$S[\phi + \Delta \phi] = S[\phi] + \int d^3x \frac{\delta S}{\delta \phi_x} \Delta \phi_x, \quad (9)$$

and substitute into (8) to get (after some algebra; see [6])

$$P[\phi'|\phi] = \frac{1}{Z} \exp \left( -\frac{1}{2} \alpha[\phi] \int d^3x (\Delta \phi_x - \Delta \bar{\phi}_x)^2 \right), \quad (10)$$

where  $Z$  is a new normalization constant and

$$\Delta\bar{\phi}_x = \langle \Delta\phi_x \rangle = \frac{1}{\alpha} \frac{\delta S}{\delta\phi_x} . \quad (11)$$

The transition probability given by (10) and (11) is the basis for dynamics:  $\phi_x$  changes by a small amount  $\Delta\phi_x = \Delta\bar{\phi}_x + \Delta w_x$  given by a drift  $\Delta\bar{\phi}_x$  plus a fluctuation  $\Delta w_x$  such that

$$\langle \Delta w_x \rangle = 0 \quad \text{and} \quad \langle \Delta w_x \Delta w_{x'} \rangle = \frac{1}{\alpha[\phi]} \delta_{xx'} . \quad (12)$$

For large  $\alpha$  the fluctuations  $\Delta w \sim O(\alpha^{-1/2})$  dominate over the drift  $\Delta\bar{\phi} \sim O(\alpha^{-1})$  which means that the trajectory in configuration space  $\mathcal{F}$  is continuous but non-differentiable — a Brownian motion. The choice of  $\Delta\lambda^2[\phi]$  or equivalently of its multiplier  $\alpha[\phi]$  is fixed by a symmetry argument. For infinitesimal  $\Delta\phi$  the dynamics is dominated by the fluctuations  $\Delta w$ . To reflect the translational symmetry of the flat configuration space  $\mathcal{F}$  we choose  $\alpha[\phi]$  so that the fluctuations in eq.(12) be independent of  $\phi$ . Therefore  $\alpha[\phi] = \text{constant}$ .

### 3 Accumulating changes: entropic time

Time is intimately related to change. In ED time is introduced as a convenient book-keeping device to keep track of the accumulation of small changes. The basic strategy, described in detail in [2][3][6], is to develop a model that includes (a) something one might identify as an “instant”, (b) a sense in which these instants can be “ordered”, (c) a convenient concept of “duration” measuring the separation between instants. This set of concepts constitutes what we will call “entropic time”. Incidentally, the model incorporates an intrinsic directionality — an evolution from past instants towards future instants. Thus, an arrow of time is generated automatically.[3]

When referring to the probability  $\rho[\phi]$  of a particular field configuration  $\phi$  it is implicit that the values  $\phi_x$  at different locations  $x$  occur at the same instant. In ED we turn this intuition around and use it to define the notion of instant: an instant  $t$  is defined by a probability distribution  $\rho_t[\phi]$ . Such instants are naturally ordered by the dynamics: if the distribution  $\rho_t[\phi]$  refers to a certain instant  $t$ , and  $P[\phi'|\phi]$  in (10) is the probability of a small change to  $\phi' = \phi + \Delta\phi$ , then we can construct the distribution

$$\rho_{t'}[\phi'] = \int D\phi P[\phi'|\phi] \rho_t[\phi] , \quad (13)$$

and use it to define what we mean by the “next” instant,  $t' = t + \Delta t$ . Thus, eq.(13) allows entropic time to be constructed, step by step, as a succession of instants. Finally, to establish a measure of duration between successive instants we consult the dynamics again — *time is defined so that motion looks simple*. We define the multiplier  $\alpha(t)$  to be independent of  $t$ ,

$$\alpha(t) = \frac{1}{\eta\Delta t} = \text{constant} , \quad (14)$$

where  $\eta$  is a constant introduced so that  $\Delta t$  has units of time. (It is further convenient to choose units so that the speed of light  $c = 1$ .) Thus, the drift velocity is

$$b_x[\phi] = \frac{\Delta\bar{\phi}_x}{\Delta t} = \eta \frac{\delta S[\phi]}{\delta\phi_x} , \quad (15)$$

and the fluctuations are given by

$$\langle\Delta w_x\rangle = 0 \quad \text{and} \quad \langle\Delta w_x\Delta w_{x'}\rangle = \langle\Delta\phi_x\Delta\phi_{x'}\rangle = \eta\Delta t\delta_{xx'} . \quad (16)$$

With this choice of  $\alpha$  the strength of the fluctuations remains constant in time. Or, in other words: *the Brownian field fluctuations constitute the standard clock that sets the scale of entropic time.*

We are now ready to study how small changes  $\Delta\phi$  accumulate as eq.(13) is iterated. The result, well known from diffusion theory (see *e.g.* [6] for details), is a functional Fokker-Planck equation which can be written as a continuity equation,

$$\partial_t\rho_t[\phi] = - \int d^3x \frac{\delta}{\delta\phi_x} (\rho_t[\phi]v_x[\phi]) , \quad (17)$$

where  $v_x[\phi]$  is the velocity of the probability flow in the  $\mathcal{F}$  space or *current velocity*,

$$v_x[\phi] = b_x[\phi] + u_x[\phi] , \quad (18)$$

and  $u_x[\phi]$  is the *osmotic velocity*

$$u_x[\phi] = -\eta \frac{\delta \log \rho_t^{1/2}}{\delta\phi_x} . \quad (19)$$

The osmotic contribution to the probability flow,  $\rho_t u_x$ , is the diffusion current in  $\mathcal{F}$  space. Since both the drift velocity  $b_x$  and the osmotic velocity  $u_x$  are gradients in  $\mathcal{F}$  space, it follows that the current velocity is a gradient too,

$$v_x[\phi] = \eta \frac{\delta\Phi[\phi]}{\delta\phi_x} \quad \text{where} \quad \Phi[\phi] = S[\phi] - \log \rho_t^{1/2}[\phi] . \quad (20)$$

## 4 Non-dissipative diffusion

The implicit constraint that the statistical manifold  $\mathcal{M}$  is rigidly fixed has led us to describe the evolution of  $\rho_t[\phi]$  as a diffusion process in  $\mathcal{F}$  space but quantum mechanics is not just diffusion. We will therefore modify this constraint by allowing  $\mathcal{M}$  to participate in the dynamics, that is, the distribution  $p_t[y|\phi]$ , its entropy  $S_t[\phi]$ , and the “phase” functional,

$$\Phi_t[\phi] = S_t[\phi] - \log \rho_t^{1/2}[\phi] , \quad (21)$$

all become time-dependent. The dynamics of  $\mathcal{M}$  is specified by imposing the conservation of a certain functional  $E[\rho_t, S_t]$  of probability and entropy that we will call the “energy”. Note that *this “energy” is an epistemic concept: it is a property not of the physical field but of our unphysical state of knowledge*. The proposed “energy” functional is chosen to be the expectation of a local density,

$$E[\rho_t, \Phi_t] = \int D\phi \rho_t[\phi] \int d^3x \mathcal{E}(\rho_t, \partial\rho_t, \Phi_t, \partial\Phi_t) . \quad (22)$$

The density  $\mathcal{E}$  is chosen so that it is invariant under time reversal and consists of the lowest non-trivial powers of the current and osmotic velocities,

$$\mathcal{E}(\rho_t, \partial\rho_t, \Phi_t, \partial\Phi_t) = \frac{\eta^2}{2} \left( \frac{\delta\Phi[\phi]}{\delta\phi_x} \right)^2 + a \frac{\eta^2}{2} \left( \frac{\delta \log \rho_t^{1/2}}{\delta\phi_x} \right)^2 + V(\phi_x, \partial\phi_x) . \quad (23)$$

The first term  $v_x^2/2$  represents “kinetic” energy.<sup>4</sup> The second term  $av_x^2/2$  represents an osmotic “potential” energy where the constant  $a$  measures its strength relative to the kinetic energy. The last term represents the more standard contribution to potential energy; in general  $V$  will depend on the field  $\phi_x$  and its spatial derivatives  $\partial\phi_x$ .

Taking the time derivative of (22), using eqs.(17), (19) and (20), after integration by parts and some algebra (eventually) yields

$$\dot{E} = \int D\phi \dot{\rho}_t \left[ \eta \dot{\Phi} + \int d^3x \left( \frac{\eta^2}{2} \left( \frac{\delta\Phi}{\delta\phi_x} \right)^2 - a \frac{\eta^2}{2} \frac{1}{\rho_t^{1/2}} \frac{\delta^2 \rho_t^{1/2}}{\delta\phi_x^2} + V \right) \right] . \quad (24)$$

Now, any instant  $t$  can be taken as the initial instant for evolution into the future. We impose that the energy  $E$  be conserved for any arbitrary choice of initial conditions, namely  $\rho_t[\phi]$  and  $\Phi_t[\phi]$ , which implies an arbitrary choice of  $\dot{\rho}_t$ . Therefore,

$$\eta \dot{\Phi} = - \int d^3x \left( \frac{\eta^2}{2} \left( \frac{\delta\Phi}{\delta\phi_x} \right)^2 - a \frac{\eta^2}{2} \frac{1}{\rho_t^{1/2}} \frac{\delta^2 \rho_t^{1/2}}{\delta\phi_x^2} + V \right) , \quad (25)$$

which we recognize as a functional form of the quantum Hamilton-Jacobi equation.

We are done. Equations (25) and the Fokker-Planck equation eq.(17) with (20),

$$\dot{\rho}_t = -\eta \int d^3x \frac{\delta}{\delta\phi_x} \left( \rho_t \frac{\delta\Phi_t}{\delta\phi_x} \right) , \quad (26)$$

are the coupled dynamical equations we seek. They describe energy conservation and entropic diffusion respectively. Eq.(25) shows how  $\rho_t$  affects the evolution of  $\Phi_t$  and eq.(26) shows how  $\Phi_t$  affects the evolution of  $\rho_t$ .

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<sup>4</sup> An overall multiplicative constant has been adjusted so the coefficient of the kinetic energy is  $1/2$ .

We can always combine the functions  $\rho_t$  and  $\phi_t$  into the complex function  $\Psi_t[\phi] = \rho_t^{1/2} \exp(i\Phi_t)$ , and rewrite eqs.(25) and (26) as a single complex equation,

$$i\eta\partial_t\Psi_t = \int d^3x \left( -\frac{\eta^2}{2} \frac{\delta^2}{\delta\phi_x^2} + V + (1-a)\frac{\eta^2}{2} \frac{1}{\rho_t} \frac{\delta^2\rho_t}{\delta\phi_x^2} \right) \Psi_t . \quad (27)$$

Next set  $\eta = \hbar$  and choose  $a = 1$  to get the functional Schrödinger equation,<sup>5</sup>

$$i\hbar\partial_t\Psi_t = \int d^3x \left( -\frac{\hbar^2}{2} \frac{\delta^2}{\delta\phi_x^2} + V(\phi_x, \partial\phi_x) \right) \Psi_t , \quad (28)$$

which concludes the derivation. At this point the potential  $V(\phi_x, \partial\phi_x)$  is essentially arbitrary. A reasonable form is obtained by doing a Taylor expansion about weak fields and gradients and then imposing the rotational and Lorentz symmetries required by the experimental evidence,

$$V(\phi_x, \partial\phi_x) = (\partial\phi_x)^2 + m^2\phi_x^2 + \lambda'\phi_x^3 + \lambda''\phi_x^4 + \dots \quad (29)$$

The various coefficients represent mass and other coupling constants. We conclude that the ED framework reproduces the standard relativistic quantum theory of scalar fields.[7]

## 5 Conclusions and discussion

Entropic dynamics provides an alternative method of quantization — entropic quantization. In the ED framework quantum field theory is a non-dissipative diffusion in the configuration space  $\mathcal{F}$ .

Entropic time is defined so that motion looks simple. In Newtonian mechanics free particles provide the clock and time is defined so that free particles cover equal distances in equal times. In the ED of fields, the field fluctuations provide the clock and entropic time is defined so that field fluctuations are uniform in space and time.

The standard predictions of quantum field theory follow immediately — just transform from the Schrödinger representation to the Heisenberg operator representation. For example, if we restrict ourselves to the first two terms in (29) we obtain the Schrödinger representation of the free Klein-Gordon field,

$$i\hbar\partial_t\Psi = \frac{1}{2} \int d^3x \left( -\hbar^2 \frac{\delta^2}{\delta\phi_x^2} + (\partial\phi_x)^2 + m^2\phi_x^2 \right) \Psi . \quad (30)$$

A standard calculation [8] of the ground state yields,

$$\Psi_t^{(0)}[\phi] = e^{-iE_0t/\hbar} \exp -\frac{1}{2} \int d^3x d^3x' \phi(\vec{x}) G(\vec{x}, \vec{x}') \phi(\vec{x}') \quad (31)$$

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<sup>5</sup>As discussed in [2] the choice  $a = 1$  does not represent any loss of generality. It can always be obtained by an appropriate rescaling of the units of  $\eta = \kappa\eta_{\text{new}}$  and regraduation of  $\Phi = \Phi_{\text{new}}/\kappa$ .



where the ground state energy,  $E_0 = \langle \int d^3x \mathcal{E} \rangle_0$ , is divergent:

$$E_0 = \frac{1}{2} \int d^3x G(\vec{x}, \vec{x}) \quad \text{where} \quad G(\vec{x}, \vec{x}') = \int \frac{d^3k}{(2\pi)^3} (\vec{k}^2 + m^2)^{1/2} e^{i\vec{k} \cdot (\vec{x} - \vec{x}')}.$$

Also, at any point  $\vec{x}$  the expected value of the field vanishes and its variance diverges,

$$\langle \phi(\vec{x}) \rangle_0 = 0 \quad \text{and} \quad \langle \phi^2(\vec{x}) \rangle_0 = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\vec{k}^2 + m^2)^{1/2}}. \quad (32)$$

Note, however, that in ED the divergent energies and variances are epistemic notions, so that once “*we free ourselves from the delusion that probabilities are physically real things*” we see that nothing physical is actually diverging.

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